ECON 4470

Problem Set 1

1. (Expected Utility)
   For each of the following utility functions, explain whether it represents a risk-averse, risk-seeking or risk neutral decision maker.
   a. $U(x) = 6x - 3x^2$
   b. $U(x) = e^x + x$

2. (CARA and CRRA utility functions)
   Formulas for risk aversion measures can be found on page 4 of handout_1_1. Formulas for utility functions can be found on page 1 of handout_1_2.
   a. Show that a CARA utility function indeed has constant rate of absolute risk aversion.
   b. Find also its rate of relative risk aversion. Is it increasing or decreasing?
   c. Show that a CRRA utility function indeed has constant relative rate of risk aversion.
   d. Find also its rate of absolute risk aversion. Is it increasing or decreasing?

3. (Quadratic Utility)
   A quadratic utility has the following form,
   $$U(x) = ax^2 + bx + c$$
   Where a, b and c are some constant numbers, with a being negative. You can take (a,b,c) = (-9,21,8) if you prefer to work with actual numbers.
   a. Remember that a quadratic utility always has a bliss point, where marginal utility from extra wealth goes from positive to negative. What value of X would give us this point? Show your steps.
   b. Find the rate of absolute risk aversion and relative risk-aversion.
   c. Show that an investor with a quadratic utility only cares about mean and variance. (Hint: Take expectation and use the formula for variance in handout 1)
   d. **[Tedious calculation]** Take (a,b,c) = (-9,21,8). Suppose the investor has $1 to invest, average stock return is 10%, risk-free return is 3% and the variance in stock return is 15%. How much money would she invest in the portfolio? (Hint: First find the average return and standard deviation faced by the investor when she invest w dollar in the risk-free asset and 1 – w in stocks. Plug the formulas into the utility function and take derivative with respect to w.)

4. (Prospect Theory)

This question asks you to think about the mug experiment discussed in class in terms of actual utility functions. Let two dimensions of choice be mugs and money, with mugs being dimension 1 and money being dimension 2. Denote outcomes in mugs and money by c1 and c2, respectively, and reference points in the two dimensions by r1 and r2, respectively. The person's utility is given by

$$4c_1 + c_2 + v(4c_1 - 4r_1) + v(c_2 - r_2)$$

where $v(x) = 0.5x$ for $x \geq 0$, and $v(x) = 2x$ for $x < 0$. You can think of the first part of the utility function $(4c_1 + c_2)$ as standard "consumption utility," and the second part $(v(4c_1 - 4r_1) + v(c_2 - r_2))$ as the reference-dependent "gain-loss utility."

a. Does this formulation capture loss aversion? Does it capture diminishing sensitivity?
b. Argue that the reference point of an owner of the mug is $r_1 = 1$ and $r_2 = 0$. Given this reference point, solve for the "selling price", the minimum price at which an owner is willing to part with her mug.
c. What is the reference point of a non-owner of a mug? Give both r_1 and r_2. Solve for the "buying price", the maximum price at which a non-owner is willing to buy a mug.
d. In a graph with mugs on the horizontal axis and money on the vertical axis, draw indifference curves for owners and non-owners. Indicate how to read off the buying and selling prices from the graph.
e. Some subjects (called the "choosers") are initially not given a mug or any money. Then, they are told that they can either have the mug or money, and are asked for the minimum amount of money which they would be willing to accept instead of the mug. Solve for this "choosing price."
f. In an actual experiment, the buying, choosing, and selling prices were found to be \$2.87, \$3.12, and \$7.12, respectively. How well does the above model explain these findings? To the extent that it cannot explain the findings, what do you think goes wrong?
g. Suppose a non-owner receives a gift of \$5 just before her buying price is elicited, and does not adjust her reference point in money to this new situation. What would her buying price be? Explain the intuition.
h. Suppose an owner loses \$5 just before her selling price is elicited, and does not adjust her reference point in money to this situation. What would her selling price be in this case?