Reference-Dependent Preferences: Examples

Alvaro has reference-dependent preferences over money. Let his realized outcome in money be c₁, and his reference point in money be r₁. Then, Alvaro's utility is given by

$$v(c_1-r_1)$$

where v(x) = x for $x \ge 0$, and v(x) = 2x for x < 0. Normalize Alvaro's initial amount of money to zero.

a. Alvaro has just agreed to take part in an experiment, and he was told by the organizers that he might receive \$10 for his participation. Suppose Alvaro can choose whether to incorporate the \$10 into his reference point. Calculate his utility with a reference point of \$0 and a reference point of \$10 if he receives \$10 for his participation. Calculate his utility with a reference point of \$0 and a reference point of \$10 if he receives \$0 for his participation. Which reference point yields higher utility? Explain the intuition.

$$(c_1 - r_1), \qquad \begin{cases} v(x) = x, for \ x \ge 0 \\ v(x) = 2x, for \ x < 0 \end{cases}$$

1)
$$(c_1, r_1) = (10,0)$$

$$v(c_1 - r_1) = 10$$
, since $c_1 - r_1 = 10 - 0 = 10 > 0$

2)
$$(c_1, r_1) = (10, 10)$$

$$v(c_1 - r_1) = 0$$
, since $c_1 - r_1 = 10 - 10 = 0 \ge 0$

3)
$$(c_1, r_1) = (0,0)$$

$$v(c_1 - r_1) = 0$$
, since $c_1 - r_1 = 0 - 0 = 0 \ge 0$

4)
$$(c_1, r_1) = (0.10)$$

$$v(c_1 - r_1) = -20$$
, since $c_1 - r_1 = 0 - 10 = -10 < 0$

Reference point of \$0 generates higher utility, since Alvaro will only experience a gain and never a loss in this case. With a reference point of \$10, Alvaro will only experience a loss and never a gain.

Now suppose Alvaro has reference-dependent preferences not just over money, but over money, mugs, and pens. Let his consumption in money, mugs, and pens be c1, c2, and c3, respectively, and let his reference point in money, mugs, and pens be r1, r2, and r3, respectively. Then, Alvaro's utility is given by

$$v(c_1-r_1) + v(4c_3-4r_2) + v(4c_3-4r_3)$$

where v(x) = x for $x \ge 0$, and v(x) = 2x for x < 0. Normalize Alvaro's initial amount of money to zero, and suppose he starts off with zero mugs and zero pens.

b. Alvaro arrives at the experiment, and it turns out that he receives an \$8 show-up fee, which he does not incorporate into his reference point for money. Suppose Alvaro is given a mug (and he does incorporate this into his reference point). Calculate his selling price by (i) writing down his reference point; and (ii) solving for the price p_s that makes him indifferent between keeping his mug and receiving nothing and giving up his mug and getting p_s .

His reference point is $(r_1, r_2, r_3) = (0,1,0)$.

Utility if Alvaro does nothing:

Initial consumption and reference points $(c_1, c_2, c_3, r_1, r_2, r_3) = (8,1,0,0,1,0)$, so $v(c_1 - r_1) + v(4c_2 - 4r_2) + v(4c_3 - 4r_4) = 8 + 0 + 0 = 8$

Utility generated if Alvaro sells the mug:

Initial consumption and reference points $(c_1, c_2, c_3, r_1, r_2, r_3) = (8 + p_s, 0, 0, 0, 1, 0)$, so $v(c_1 - r_1) + v(4c_2 - 4r_2) + v(4c_3 - 4r_4) = 8 + p_s - 8 = p_s$

Alvaro's selling price for the mug is given by equating utility in the two cases:

$$p_{\rm c} = 8$$

So Alvaro is willing to sell the mug for \$8, to remain his utility unchanged.

c. Now suppose Alvaro is not given a mug. Calculate his buying price by (i) writing down his reference point; and (ii) solving for the price p_B such that he is indifferent between getting a mug for p_B and not getting or paying anything.

The question does not make it clear whether Alvaro expects a mug or not. Base on the wording of b., we assume Alvaro only expects to have a mug if he is given one, so his reference point in this situation is $(r_1, r_2, r_3) = (0,0,0)$.

Utility if Alvaro does nothing:

Initial consumption and reference points $(c_1, c_2, c_3, r_1, r_2, r_3) = (8,0,0,0,0,0)$, so $v(c_1 - r_1) + v(4c_2 - 4r_2) + v(4c_3 - 4r_4) = 8 + 0 + 0 = 8$

Utility generated if Alvaro buys one mug:

Consumption and reference points are $(c_1, c_2, c_3, r_1, r_2, r_3) = (8 - p_B, 1,0,0,0,0)$, so

$$v(c_1 - r_1) + v(4c_2 - 4r_2) + v(4c_3 - 4r_4) = 8 - p_B + 4 + 0 = 4 - p_B$$

The buying price is given by equating utility in the two cases:

$$4 - p_B = 8$$
$$p_B = 4$$

So Alvaro is willing to pay \$4 for one mug, to remain his utility unchanged.

d. Argue briefly that Alvaro's buying and selling prices for pens are the same as for mugs.

Since the utility function of pens is identical to his utility function for mugs, the buying and selling price would be the same under same reference point conditions

For the rest of the question, suppose that Alvaro has two options in the experiment: to buy a mug for \$7, and to buy a pen for \$6.

e. Suppose Alvaro arrives at the experiment with a reference point of having one mug and one pen (perhaps because he was told he could get these as gifts). His reference point in money is still \$0 and he still receives the \$8 show-up fee. Would he buy a mug for \$7 if that was the only choice he was offered? Would he buy a pen for \$6 if that was the only choice he was offered? Would he make both purchases? Briefly explain the intuition.

His reference point will be $(r_1, r_2, r_3) = (0,1,1)$.

Utility if Alvaro does nothing:

Initial consumption and reference points
$$(c_1, c_2, c_3, r_1, r_2, r_3) = (8,0,0,0,1,1)$$
, so $v(c_1 - r_1) + v(4c_2 - 4r_2) + v(4c_3 - 4r_4) = 8 - 8 - 8 = -8$

If he spends \$7 on the mug:

$$v(c_1 - r_1) + v(4c_2 - 4r_2) + v(4c_3 - 4r_4) = 1 + 0 - 8 = -7$$

If he spends \$6 on the pen:

$$v(c_1 - r_1) + v(4c_2 - 4r_2) + v(4c_3 - 4r_4) = 2 - 8 + 0 = -6$$

If he spends \$13 on one mug and one pen:

$$v(c_1 - r_1) + v(4c_2 - 4r_2) + v(4c_3 - 4r_4) = -10 + 0 + 0 = -10$$

In summary, Alvaro is willing to get either one mug or one pen but not both. What matters here are not the budget constraint, but his gains and losses: As long as Alvaro uses less than \$8, he is forgoing an expected gain in money, which is not too painful. Buying too much, however, will result in Alvaro using more than \$8, which brings to into the loss region for money, which is very painful.

f. Now suppose Alvaro can choose whether to enter the above experiment with a reference point of getting the mug or a reference point of getting the pen, knowing the prices in advance. (He still cannot choose his reference point in money, which is \$0, and he knows he will get \$8.) Which one would he choose? Briefly explain the intuition.

If he decides to expect neither pen nor mugs from the experiment, his reference point will be $(r_1, r_2, r_3) = (0,0,0)$

Utility if Alvaro does nothing:

Initial consumption and reference points $(c_1, c_2, c_3, r_1, r_2, r_3) = (8,0,0,0,0,0)$, so

$$v(c_1 - r_1) + v(4c_2 - 4r_2) + v(4c_3 - 4r_4) = 8 - 0 - 0 = 8$$

If he spends \$7 on the mug:

$$v(c_1 - r_1) + v(4c_2 - 4r_2) + v(4c_3 - 4r_4) = 1 + 4 + 0 = 5$$

If he spends \$6 on the pen:

$$v(c_1 - r_1) + v(4c_2 - 4r_2) + v(4c_3 - 4r_4) = 2 + 0 + 4 = 6$$

If he spends \$13 on one mug and one pen:

$$v(c_1 - r_1) + v(4c_2 - 4r_2) + v(4c_3 - 4r_4) = -10 + 4 + 4 = -2$$

So he will not buy any of mugs or pen at the end.

If he decides to expect to get a mug from the experiment, his reference point will be $(r_1, r_2, r_3) = (0,1,0)$.

Utility if Alvaro does nothing:

Initial consumption and reference points $(c_1, c_2, c_3, r_1, r_2, r_3) = (8,0,0,0,1,0)$, so

$$v(c_1 - r_1) + v(4c_2 - 4r_2) + v(4c_3 - 4r_4) = 8 - 8 + 0 = 0$$

If he spends \$7 on the mug:

$$v(c_1 - r_1) + v(4c_2 - 4r_2) + v(4c_3 - 4r_4) = 1 + 0 + 0 = 1$$

If he spends \$6 on the pen:

$$v(c_1 - r_1) + v(4c_2 - 4r_2) + v(4c_3 - 4r_4) = 2 - 8 + 4 = -2$$

If he spends \$13 on one mug and one pen:

$$v(c_1 - r_1) + v(4c_2 - 4r_2) + v(4c_3 - 4r_4) = -10 + 0 + 4 = -6$$

So he will buy a mug but not a pen at the end. Similarly, he will buy a pen but not a mug if he incorporates only a pen into his reference point at the beginning.

Intuition: Pain from not getting what one expects is much more than the utility gain from getting the expected. Because people are willing to pay a lot to avoid an unexpected loss, they have a strong tendency to do what they expect themselves to do. Expectations can thus be a self-fulfilling prophecy—if I expect myself buying, I buy, but if I expect myself not buying, I do not buy.

If people are aware of the self-fulfilling prophecy and can really choose their reference point as the question suggests, they should choose the one the give them the highest utility at the end. In this case, Alvaro should choose not to expect anything.