University of California, Berkeley — Spring 2008
ECON 100A Section 109, 112

Game Theory 2

The type of games in which we represent with a single matrix is called *single-period normal form game*. This handout will deal with two additional forms of games: repeated games and extensive form games.

I. Repeated Games

When a game is played repeatedly, it is called a repeated game. Repeated games often allows for radically different equilibriums than single period games. In this course we learn one particular example—the infinitely repeated prisoners' dilemma.

Infinitely Repeated Prisoners' Dilemma

Consider the following game:

		Player 2	
		C	D
Player 1	C	2,2	0,3
	D	3,0	1,1

(D,D) is the Nash Equilibrium if the game is played only once. If the game is played repeatedly infinite rounds, however, the follow strategy might hold in equilibrium:

For each player, choose
- C if the other player has been choosing C in all previous rounds
- D Otherwise

What this strategy does is to reward the other player's cooperation with further cooperation, and punish defection with eternal defection. Under this strategy by defecting a player obtains an immediately increase in payoff –getting 3 instead of 2—since punishment does not come in till next round. Starting from the next round, however, the player losses—she would get 1 instead 2 in each subsequent round. Defect would not occur if the immediately gain is smaller than the discounted future loss. Therefore if the interest is r, there will be no defection if

$$\textbf{immediate benefit} < \left(\frac{1}{1-r}\right) \textbf{loss in next period} + \left(\frac{1}{1-r}\right)^2 \textbf{loss in the period after the next} \ldots$$

$$\mathbf{3-2<\left(\frac{1}{1-r}\right)(2-1)+\left(\frac{1}{1-r}\right)^{2}(2-1)+\cdots}$$

$$\mathbf{1<\frac{1}{1-\left(\frac{1}{1-r}\right)}-1}$$

$$\mathbf{1<\frac{1}{r}}$$

As interest rate goes up the right hand side becomes smaller, which means a smaller immediate benefit is enough to induce cheating. *Thus in an infinitely repeated prisoners' dilemma, a higher interest rate makes cheating more likely*. Intuitively when interest rate is high the present value of future payoff is relatively small, making the immediately benefit of cheating more attractive.

Note that this punishment strategy only works if the game is played infinitely. If the game stops at a certain round *T*, players would surely defect at that round, since there is no subsequent round for punishment to come into play. This removes the punishment for cheating in round *T – 1*, so players would also cheat in round *T – 1*. Continue with this logic and we can see that players would cheat in all rounds.

II. Extensive Form Games

Normal form is ideal for displaying simultaneous move games, but does not for games where players move sequentially. In these cases we use the *extensive form*. Extensive form is simply a tree diagram, with each node being a player and each branch an action. The lower a player is on the tree, the later she moves. At the bottom of the tree are payoffs.

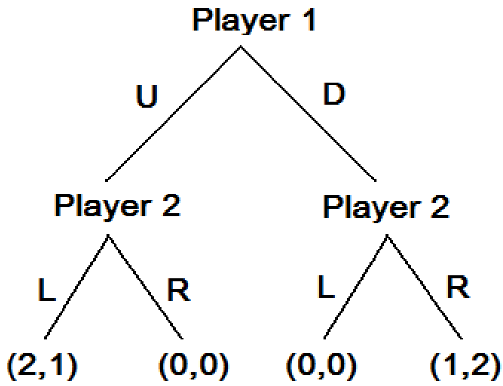
<u>Example</u>

Consider the following game in normal form:

		Player 2	
		L	R
Player 1	U	2,1	0,0
	D	0,0	1,2

Remember we found that (U,L) and (D,R) are both Nash Equilibria.

Now suppose Player 1 moves first, we can represent the game in extensive form,



Equilibrium in an extensive form game is called *Subgame Perfect Equilibrium*. As we shall see, SPE are not the same as pure strategy Nash Equilibria in the normal form game.

III. First Mover Advantage

What would Player 1 do in the above game? From the extensive form representation we can see that if player 1 goes *U*, Player 2 would go *L*, giving Player 1 a payoff of 2. On the other hand if Player 1 goes *D* Player would go *R*, giving Player 1 a payoff of only 1. Thus Player 1 would go *U* and not *D*. So in contrast to the simultaneous move version of the game, (D,R) is not an equilibrium.

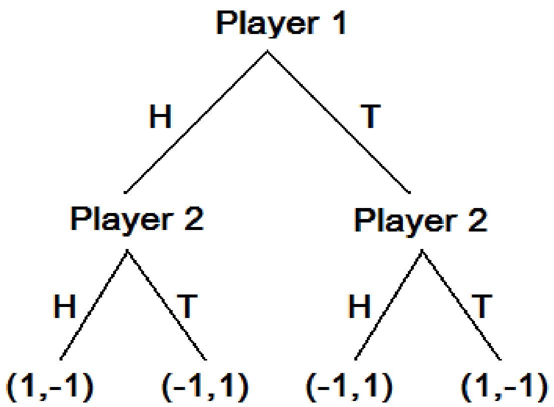
This is the *first mover advantage—when there is more than one Nash Equilibria, the Player would moves first can pick the one that gives her the highest payoff.*

IV. **Second Mover Advantage**

Consider the following game:

		Player 2	
		H	T
Player 1	H	1,-1	-1,1
	T	-1,1	1,-1

Recall that this game has no pure strategy Nash Equilibrium. Now suppose Player 1 moves first,



If Player 1 chooses *H* Player 2 would choose *T*. If Player 1 chooses *T* instead Player 2 would choose *H*. In both cases Player 1 gets a payoff of -1 while Player 2 gets a payoff of 1.

This is the *second mover advantage—when there is no Nash Equilibrium, the Player who moves later can improve her payoff relative to the simultaneous version of the game*.

What is happening is in the sequential move version of the game Player 1 has to commit to one action before Player 2 chooses her action. In the simultaneous move version, on the other hand, Player 1 can and will always switch her action given what Player 2 does.