

Auctions

Two things to note:

1. Throughout this section I will assume that there are only two bidders. This assumption is not all that unrealistic because for the types of auction we study, what really matters ultimately is the two highest bidders.
2. We will use logic rather than math to study the scenarios. The reason is, well, the math is just too hard...

I. Basics

Bidders: 1 and 2

Valuations: v_1 and v_2

- Valuations are how much the good worth to each bidder. Each bidder knows her own valuation but not others'.

Bidding Strategies: $b_1(v_1)$ and $b_2(v_2)$

- Bidding strategies are functions of valuations—given my valuation, my bid strategy will tell me what bid maximizes my expected utility. Unlike valuations, the bidding strategy of every bidder is common knowledge. Not that bad an assumption because we assume all bidders are identical except for their valuations.

Without loss of generality we shall assume the row of bidder 1 in our analyses. Our goal: get as large a gain as possible—in other words maximize $v_1 - b_1$.

II. English Auction

Open cry: The auctioneer publicly announces higher and higher prices until there is only one bidder left.

Suppose $v_1 > v_2$. What would happen? What should bidder 1 do?

Suppose instead $v_1 < v_2$. What would happen now and what should bidder 1 do?

III. Second Price Auction

Sealed bid: Each bidder submits their bid in secrecy—each bidder is not sure what her opponents are bidding.

Second Price: The highest bidder pays the second highest bidder's bid.

Question: Should bidder 1 post a bid lower, higher or equal to her valuation?

Possible Actions:

1. $b_1 < v_1$

What happens if $b_1 < b_2 < v_1$?

2. $b_1 > v_1$

What happens if $v_1 < b_2 < b_1$?

3. $b_1 = v_1$

$b_1 < b_2 \Leftrightarrow v_1 < b_2 \rightarrow$ losses and pays nothing

$b_1 > b_2 \Leftrightarrow v_1 > b_2 \rightarrow$ wins and pay b_2

IV. First Price Auction

Sealed bid: Each bidder submits their bid in secrecy.

First Price: The highest bidder pays her bid.

1. $b_1 > v_1$

What happens if $v_1 < b_2 < b_1$?

2. $b_1 = v_1$

$b_1 < b_2 \Leftrightarrow v_1 < b_2 \rightarrow$ losses and pays nothing

$b_1 > b_2 \Leftrightarrow v_1 > b_2 \rightarrow$ wins and pay b_1

What happens if we lower b_1 a little?

Question: How much should bidder 1 lower her bid?

Answer: It depends on the distribution valuations and utility functions.

Example

2 risk neutral bidders. Valuations uniformly distributed on $[0,10]$

V. Revenue Equivalence

When each bidder's valuation is known only to herself, English Dutch, First Price and Second Price auctions results in the same revenue for the auctioneer.

Idea: Rational bidders take into consideration how her payoff is affected in each form of auction.

Question to ponder: Bidders rational in real world?