

## Competitive Equilibrium

### I. Perfectly Competitive Market Equilibrium

How is the market price in a perfectly competitive market determined? To tackle that we first have to lay out the assumptions of the model:

*A1. Price Taking*

*A2. Product Homogeneity*

*A3. Free Entry and Exit with infinitely many potential entrants*

A1 gives us the supply curve for each individual seller. A2 tells us that they are selling in the exact same market. Finally, A3 tells us that all sellers earn zero profit in long run equilibrium—because any level of positive profit would induce new entrants to the market, raising total output and thereby reducing market price; this process stops only when profit drops to zero.<sup>1</sup> To make the math easy we usually assume the follow too

*A4. Sellers have identical cost functions*

#### A. Short Run

1. Short run cost functions
2. Number of sellers is fixed

Short run has dual implications here—except that for cost it also means the number of sellers is fixed. Fixing the number of sellers at  $N$ , we have two unknowns  $p$ ,  $Q$  and two equations:

$$\begin{cases} Q_D(p) & \text{Market Demand} \\ Q_S(p) = N \cdot q(p) & \text{Market Supply} \end{cases}$$

Solving  $Q_D(p) = Q_S(p)$  gives us the short run equilibrium price and quantity.

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<sup>1</sup> Technically the process stops when the marginal seller—the seller in production that has the highest cost—earns zero profit. This distinction is important if we allow the sellers to have different costs. Any more profitable sellers would have entered already and could earn positive profit, even in the long run.

## B. Long Run

1. Long run cost functions
2. Number of sellers is flexible

In the long run the number of firms can be anything. Remember assumption A3 tells us that in the long run sellers should earn zero profit. Now long run profit for an individual seller is

$$\begin{aligned}\pi^{LR}(p) &= R(q) - C^{LR}(q) \\ &= AR(q) \cdot q - AC^{LR}(q) \cdot q \\ &= [AR(q) - AC^{LR}(q)] \cdot q \\ &= [p - AC^{LR}(q)] \cdot q\end{aligned}$$

The trick is to solve for the  $q$  such that  $AC^{LR}(q)$  is minimized. This is given by

$$\frac{d}{dq} AC^{LR}(q) = 0 \quad \text{or} \quad AC^{LR}(q) = MC^{LR}(q)$$

This gives us the value of long run equilibrium individual supply  $q^*$  and minimized long run average cost  $AC^{LR}(q^*)$ . We then pick  $p$  so that  $\pi(p) = 0$ . There are three possible cases here:

- i. If  $AC^{LR}(q)$  is U-shaped

$$\begin{aligned}p^* &= AC^{LR}(q^*) \\ N &= \frac{Q_D(p^*)}{q^*} \\ Q_S &= N \cdot q^* = Q_D(p^*)\end{aligned}$$

Remember  $N$  is the number of firms.

- ii. If  $AC^{LR}(q)$  is strictly upward sloping

$$\begin{aligned}p^* &= AC^{LR}(0) \\ N &= \infty \\ Q_S &= Q_D(p^*)\end{aligned}$$

- iii. If  $AC^{LR}(q)$  is strictly downward sloping

$$\begin{aligned}N &= 1 \\ \left\{ \begin{array}{l} Q_S = q(p) \\ Q_D(p) \end{array} \right.\end{aligned}$$

When there is strictly downward-sloping long run average cost it is most efficient for one single seller to dominate the market, which is monopoly. It is hard to justify the competitive market assumptions in this case.

Note that the long run market supply always equals to one single value, so *it is a horizontal line in all cases*. LR-equilibrium price is always the same; any shift in demand is compensated by the number of sellers in the long run. This is true unless we assume that long run costs depend on the number of firms  $N$ .

## II. Non-perfectly-elastic Long Run Supply Curve

We tend to think that market supply is upward slopping in reality, even in the long run; we can achieve this by assuming that sellers' cost functions change with

1. Number of firms in the market, or
2. Aggregate output.

We get an upward slopping market supply curve if sellers' costs increase with 1 or 2; we get a downward slopping market supply curve if the opposite is true.

## III. Example

Consider the market for ice cream cones in a small town. There are many potential ice cream vendors. Demand for ice cream cones is given by  $Q(P) = 126 - P$ .

(a) Suppose all the potential ice cream vendors have identical long run cost functions given by

$$C^{LR}(q) = 20 + 6q + 5q^2$$

Find the long-run equilibrium price, market quantity, number of firms and the amount produced by each firm.

(b) Suppose short run cost when deviating from a long run output  $q^{LR}$  is

$$C^{SR}(q) = 20 + 6q + 5q^2 + (q - q^{LR})^2$$

Write down the short run cost function for the long run equilibrium in part (a).

(c) Suppose demand surged to  $P(Q) = 120 - 2Q$ . Find the short run equilibrium price, market quantity, number of firms and amount produced by each firm.

(d) Find the long run equilibrium price, market quantity, number of firms and amount produced by each firm for the demand in part (d).

(e) Now suppose there are only 5 firms that have access to locally produced milk and therefore have the ability to produce at the cost structure given in part a. The remaining potential vendors must buy milk from further away and thus have costs given by  $C(q) = 15 + 10q + 15q^2$ . Find the new long-run equilibrium price, quantity, number of each type of firm and the amounts they produce.