University of California, Berkeley Spring 2008
ECON 100A Section 109, 112

Profit Maximization and Competitive Supply

A producer/seller/firm ultimately aim is neither product maximization nor cost minimization but profit maximization.

I. Basic Setting

Profit Function

Profit is given by the profit function $\pi(q)$.

$$\pi(q) = R(q) - C(q)$$

Where $R(q)$ is total revenue from sales and $C(q)$ is total cost. Notice that the profit function depends only on *q*—we do not really care about inputs anymore. This should not be surprising since we get the total cost function from cost minimization, a topic we just went over.

Total Cost Function

Generally questions on profit maximization will give you a $C(q)$ to start with. If we use the short run cost function as $C(q)$ we get short run profit, and similarly for long run:

$$\pi^{SR}(q) = R(q) - C^{SR}(q)$$
$$\pi^{LR}(q) = R(q) - C^{LR}(q)$$

Short run and long run costs are crucial in the determination of competitive market equilibrium.

Total Revenue Function

Most dynamic in profit maximization enters from the total revenue function. We shall discuss two basic market settings here: perfect competition and monopoly.

II. Profit Maximization Behavior

Condition 1: *MR = MC*

No matter what market settings we are in, a seller's aim is to maximize the profit function:

$$\max_q \{\pi(q)\}$$

Taking the first order condition,

$$\frac{d\pi(q)}{dq} = 0$$

$$\frac{d}{dq}\left[R(q) - C(q)\right] = 0$$

$$\frac{d}{dq}R(q) - \frac{d}{dq}C(q) = 0$$

$$\frac{d}{dq}R(q) = \frac{d}{dq}C(q)$$

$$\boxed{MR = MC} \qquad (1)$$

The profit maximizing condition is *marginal revenue equals marginal cost*—this should not be surprising to us. Marginal cost we are familiar in deriving; we will consider marginal revenue in a moment.

Condition 2: (Not) Shutting-down Condition

<u>Short Run</u>

Product output *q* only if

$$\boxed{AR(q) \geq AVC^{SR}(q)} \qquad (2)$$

<u>Long Run</u>

If profit drops below zero a seller is better off shutting down, so a seller would produce output only if

$$\pi^{LR}(q) \geq 0$$

This is often expressed in terms of average revenue and average (total) cost.

$$\boxed{AR(q) \geq AC^{LR}(q)} \qquad (3)$$

$AVC^{SR}(q)$ can be higher or lower than $AC^{LR}(q)$; a seller could temporarily shut down in the short run but reopen in the long run if the profit-maximizing output *q* satisfies

$$AC^{LR}(q) \leq AR(q) < AVC^{SR}(q)$$

or do the opposite if

$$AVC^{SR}(q) \leq AR(q) < AC^{LR}(q)$$

A. Perfect Competition

This is the situation that we focus on in chapter 8 and 9. Related to profit maximization the punch line is *a seller in a perfectly competitive market takes the price of its output as given*. So total revenue for the seller is

$$R(q) = p \cdot q$$

where p is the *market price*, taken as given by the seller. How the market price is determined we shall deal with in a moment; let us first find marginal revenue,

$$MR(q) = \frac{d}{dq}(p \cdot q) = p$$

Notice that average revenue is also p_m, since

$$AR(q) = \frac{R(q)}{q} = p$$

Individual Supply Curve

Combining the above with condition 1 and 2 we have the *supply curve* for the seller:

- $p = MC$ gives us the profit maximizing q at each level of p, while
- The price level at which the seller will start selling is given by

$$p > AVC^{SR}(q) \qquad \text{in short run}$$
$$p > AC^{LR}(q) \qquad \text{in long run.}$$

Market Supply Curve

The *market supply curve* is the horizontal summation of all individual supply curves. In other words, the market output for a given price is the sum of individual outputs at that price. Let $Q(p)$ be the market output and $q_i(p)$ output of seller i. If there are N sellers then

$$Q_S(p) = \sum_{i=1}^{N} q_i(p)$$