2. A farmer in Ghana has a plot of land and can plant either Cassava or Pineapples. Given a good year of rain, pineapples are much more profitable than Cassava. However, in a bad year of rain Pineapples do much worse than Cassava. Assume that the farmer has no access to insurance and has to make the planting decision before he knows the outcome of rain. The pay-off matrix and probability of good/bad year of rain is given below.

Table 1: Farmer's Pay-Off Matrix per Plot

| | Good Rain Year | Bad Rain Year |
|---|---|---|
| Pineapple | 121 | 25 |
| Cassava | 64 | 49 |
| Probability | 1/3 | 2/3 |

a. Calculate the expected pay-off and variance for each type of crop.
b. Which does the farmer plant if the farmer's utility for income is given by $U(I) = I^{1/2}$? What the farmer's risk attitude?
c. Which does the farmer plant if the farmer's utility for income is given by $U(I) = I^{2}$? What the farmer's risk attitude?

Consider $U(I) = I^{1/2}$ again,

d. What is the risk premium if farmer has to plant pineapple? Plant Cassava?
e. Would the farmer plant pineapple if the government provides 50 in subsidy if he does so?

*a. Calculate the expected pay-off and variance for each type of crop.*

For pineapple,

$$E[X] = \sum_i \Pr_i \cdot X_i$$
$$= \frac{1}{3}\cdot 121 + \frac{2}{3}\cdot 25 = \frac{171}{3}$$
$$Var(X) = E[X^2] - E(X)^2$$
$$= \left(\frac{1}{3}\cdot 121^2 + \frac{2}{3}\cdot 25^2\right) - \left(\frac{171}{3}\right)^2 = 2048$$

For cassava,

$$E[X] = \sum_i \Pr_i \cdot X_i$$
$$= \frac{1}{3}\cdot 64 + \frac{2}{3}\cdot 49 = \frac{162}{3}$$
$$Var(X) = E[X^2] - E(X)^2$$
$$= \left(\frac{1}{3}\cdot 64^2 + \frac{2}{3}\cdot 49^2\right) - \left(\frac{162}{3}\right)^2 = 50$$

*b. Which does the farmer plant if the farmer's utility for income is given by U(I) = $I^{1/2}$? What the farmer's risk attitude?*

For pineapple,

$$E[U(X)] = \sum_i \Pr_i \cdot U(X_i)$$
$$= \frac{1}{3} \cdot 121^{1/2} + \frac{2}{3} \cdot 25^{1/2} = \frac{21}{3}$$

For cassava,

$$E[U(X)] = \frac{1}{3} \cdot 64^{1/2} + \frac{2}{3} \cdot 49^{1/2} = \frac{22}{3}$$

Since expected utility from cassava is higher than that from pineapple, the farmer will plant cassava.

Taking the bet of 2 with Pr = 1/2 and 0 with Pr=0

$$E[U(X)] = \frac{1}{2} \cdot 2^{1/2} + \frac{1}{2} \cdot 0^{1/2} = \frac{1}{\sqrt{2}}$$

While 1 for sure gives

$$U(1) = 1^{1/2} = 1$$

Since $E[U(X)] < U(1)$ the farmer is risk averse.

*c. Which does the farmer plant if the farmer's utility for income is given by U(I) = $I^2$? What the farmer's risk attitude?*

For pineapple,

$$E[U(X)] == \frac{1}{3} \cdot 121^2 + \frac{2}{3} \cdot 25^2 = 5297$$

For cassava,

$$E[U(X)] = \frac{1}{3} \cdot 64^2 + \frac{2}{3} \cdot 49^2 = 2966$$

Since expected utility from cassava is lower than that from pineapple, the farmer will plant pineapple.

Taking the bet of 2 with Pr = 1/2 and 0 with Pr=0

$$E[U(X)] = \frac{1}{2} \cdot 2^2 + \frac{1}{2} \cdot 0^2 = 2$$

While 1 for sure gives

$$U(1) = 1^2 = 1$$

Since $E[U(X)] > U(1)$ the farmer is risk loving.

*Consider U(I) = $I^{1/2}$ again,*

*d. What is the risk premium if farmer has to plant pineapple? Plant Cassava?*

We find risk premium *r* using the formula $E[U(X)] = U(E[X] - r)$

For pineapple,

$$\frac{21}{3} = U\left(\frac{171}{3} - r\right)$$

$$\frac{21}{3} = \left(\frac{171}{3} - r\right)^{1/2}$$

$$r = 8$$

For cassava,

$$\frac{22}{3} = U\left(\frac{162}{3} - r\right)$$

$$\frac{22}{3} = \left(\frac{162}{3} - r\right)^{1/2}$$

$$r = 0.22$$

The larger risk premium for pineapple comes from the higher risk.

*e. Would the farmer plant pineapple if the government provides 50 in subsidy if he does so?*

The payoff for pineapple is now 171 in a good year and 75 in a bad year. So

$$E[U(X)] = \frac{1}{3} \cdot 171^{1/2} + \frac{2}{3} \cdot 75^{1/2} = 10.13 \approx \frac{30}{3}$$

Since the expected utility from pineapple is now higher than that of cassava (30/3 > 22/3) the farmer will plant pineapple.