University of California, Berkeley Spring 2007

ECON 100A Section 115, 116

Choice under Uncertainty

In the real world there is plenty of uncertainty—investment, accident, gamble are a few examples. How does a rational individual choose under uncertainty? This part of the course attempts to provide a basic framework to answer this question.

I. Basics Probability

Let *i* be some event, $\Pr_i$ be its probability of happening and X_i its payoff.

$$\text{Expected Value:} \quad E[X] = \sum_i \Pr_i \cdot X_i$$

$$\text{Variance:} \quad Var(X) = E[X^2 - E(X)]$$

$$= E[X^2] - E(X)^2$$

$$\text{Standard Deviation:} \quad \sigma_x = \sqrt{Var(X)}$$

In economics we often describe situations with uncertainty as a bet/lottery. Expected value is also called the *mean*. In general $\Pr_i$ and X_i are functions of *i*; in this class the value of $\Pr_i$ is usually given while X_i often needs to be calculated.

II. Expected Utility Theory

In order to compare different choices under uncertainty we need a way to summarize their uncertain payoffs into a certain number, just like how we went from preferences to utility previously. The fundamental theory on this is *expected utility theory* (hereon EUT)

Let *i* be some event, $\Pr_i$ be its probability of happening and X_i its payoff. The *expected utility* is defined as:

$$E[U(X)] = \sum_i \Pr_i \cdot U(X_i)$$

Suppose facing with two situations with uncertainty with payoffs X_i and Y_i. EUT states that the individual would prefer the first to the second if

$$E[U(X)] > E[U(Y)]$$

In this class you are basically going to be asked to do just that.

Note the difference between expected *payoff* and expected *utility*,

Expected Wealth / Payoff : $E[X] = \sum_i \Pr_i \cdot X_i \quad = \Pr_1 \cdot X_1 + \Pr_2 \cdot X_2 + ...$

Expected Utility : $E[U(X)] = \sum_i \Pr_i \cdot U(X_i) \quad = \Pr_1 \cdot U(X_1) + \Pr_2 \cdot U(X_2) + ...$

(Remember *i* be some event, $\Pr_i$ be its probability of happening and X_i its payoff/wealth)

An agent always chooses the option that gives the highest expected utility, not expected payoff

III. Risk Attitudes

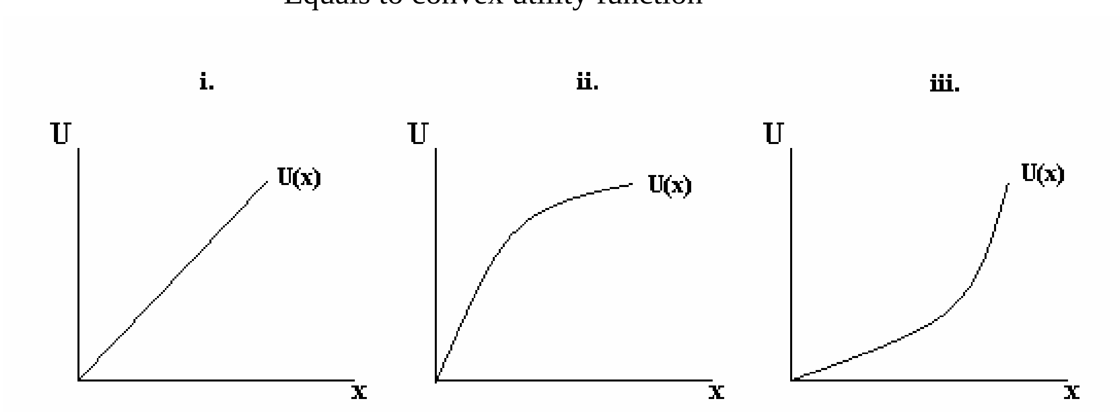
The risk attitudes we learn in this course represent the earliest attempts from economists to model the different behaviors under risk. While insightful they cannot explain many phenomena, such as the purchase of insurance while gambling.

<u>Definitions</u>

i. Risk Neutral
   - Benchmark "rational" case; usual assumption on firms
   - Equals to linear utility function

ii. Risk Averse
   - Usual assumption on consumers
   - Equals to concave utility function

iii. Risk Loving
   - Gamblers, etc.
   - Equals to convex utility function



Distinguish Between Different Risk Attitudes with a Fair Bet

We can easily distinguish between different risk attitudes by using a fair bet. Take a bet that pays 2 with probability of 1/2 and 0 otherwise—this makes calculation easy—and compare it to getting 1 for sure; we have

i. Risk Neutral $E[U(X)] = \frac{1}{2}U(2) + \frac{1}{2}U(0) = U(1)$

ii. Risk Averse $E[U(X)] < U(1)$

iii. Risk Loving $E[U(X)] > U(1)$

Risk Neutral

A risk neutral person only cares about the expected payoff/wealth $E[X]$; she does not care about risk at all.

Risk Averse and Risk Loving

If you know the choice of a risk neutral person then you know the following *even without the utility function*:

Risk neutral person...	Risk adverse person…	Risk loving person…
Takes the bet	No idea	Takes the bet for sure
Does not take the bet	Does not take the bet for sure	No idea

IV. Risk Premium and Maximum Insurance Payment

<u>Risk Premium</u>

Risk Premium is the horizontal difference between the expected utility from a bet and the utility function *at the same utility level*.

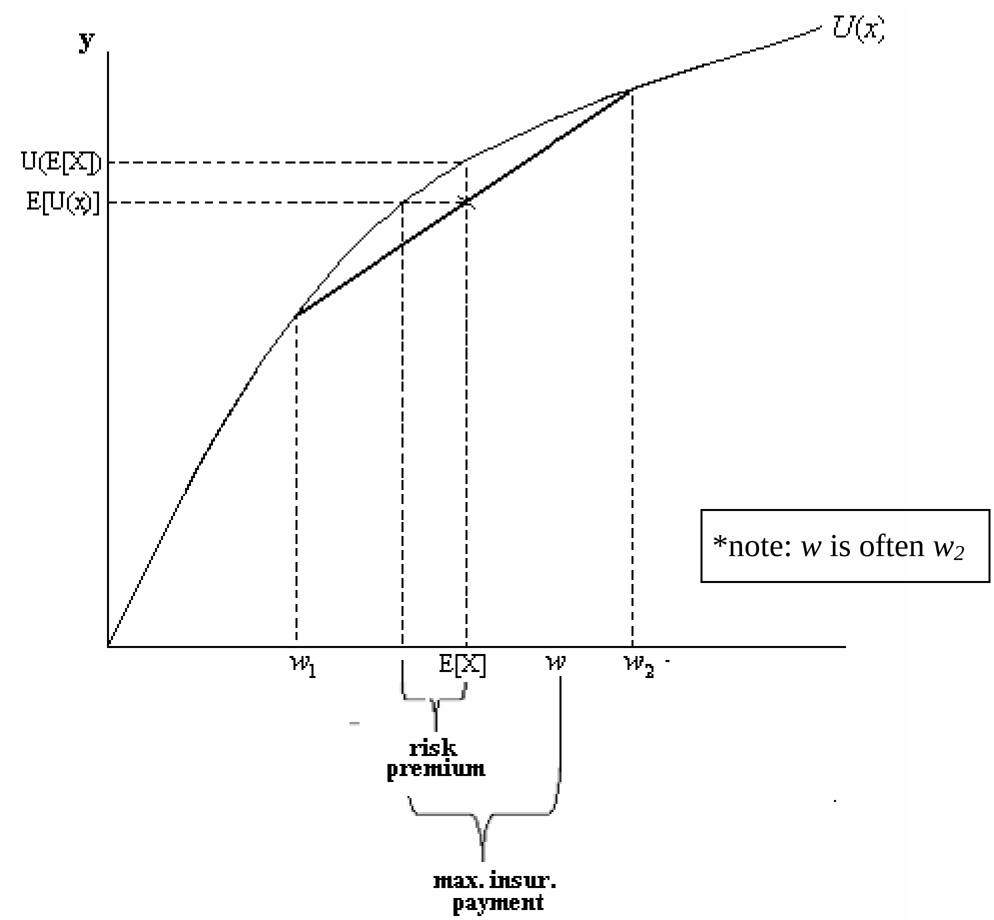
Let r be the risk premium, r can be found from the following formula:

$$E[U(X)] = U(E[X] - r)$$

<u>Maximum Insurance Payment</u>

Let w be the initial wealth and r be the risk premium, the maximum amount willing to pay for insurance is

$$w - E[X] + r$$



Examples

1. You are spending time with your friend Juanita (she loves to gamble). You tell her you have 25 on you and she proposes a bet. She'll flip a fair coin and if you call it correctly she'll owe you $11, but if not you owe her $10. Your utility for money is given by $U(M) = M^{1/2}$, and Juanita's utility for money is given by $U(M) = M^{5/4}$.

   (a) Do you accept the gamble?
   (b) What if you would only owe her $7, would you make the gamble?
   (c) What is the risk premium in each of the cases?

2. A farmer in Ghana has a plot of land and can plant either Cassava or Pineapples. Given a good year of rain, pineapples are much more profitable than Cassava. However, in a bad year of rain Pineapples do much worse than Cassava. Assume that the farmer has no access to insurance and has to make the planting decision before he knows the outcome of rain. The pay-off matrix and probability of good/bad year of rain is given below.

Table 1: Farmer's Pay-Off Matrix per Plot

	Good Rain Year	Bad Rain Year
Pineapple	121	25
Cassava	64	49
Probability	$1/3$	$2/3$

The farmer's utility for income is given by: $U(I) = I^{1/2}$.

   (a) What is the expected pay-off for each type of crop?
   (b) Which does the farmer plant? (Must answer why)
   (c) Now, assume that the government would like more farmers to plant Pineapples. They create an insurance mechanism that would ensure the income of Pineapples. What is the maximum premium they can charge that would induce this farmer to plant Pineapples?
   (d) Under the above insurance scheme, what should the government expect to pay?
   (e) Now assume the farmer's neighbor invented a device that could perfectly predict the weather, thus giving the farmer perfect information on what weather to expect. Up to what amount would the farmer be willing to pay for the device, i.e. what is the value of information?