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ECON 100A Section 115, 116

Oligopoly 2

In all the following cases let the inverse demand be $P_D(Q) = a - bQ$, where $Q = q_1 + q_2$ is the total output. We shall assume that the sellers have identical marginal cost = c.

I. Cournot with more than two firms

Remember in the two firms, identical marginal cost case the reaction function for the first Cournot firm is

$$q_1 = \frac{a - bq_2 - c}{2b}$$

If we start with n firms instead, we will instead get

$$q_1 = \frac{a - b(q_2 + ... + q_n) - c}{2b}$$

The reaction for the remaining n - 1 firms are similar. Actually solving n equations are tedious to say the least, so what we do is we note that all the firms are identical and solve for the *symmetric equilibrium*—in other words, we look for the solution where $q_1 = q_2 = ... = q_n = q$. With this the n equations become one equation,

$$q = \frac{a - b(n-1)q - c}{2b}$$

rearranging gives us the solution

$$q = \frac{a - c}{(n+1)b}$$

Price is then

$$\begin{aligned}
&P_D(nq) \\
&= a - bn\left[\frac{a - c}{(n+1)b}\right] \\
&= a - \frac{n(a-c)}{n+1} \\
&= \frac{a}{n+1} + \frac{nc}{n+1}
\end{aligned}$$

If n = 1 we have monopoly price; as n→∞ we have price→marginal cost as in perfect competition.

II. Residual Demand in Cournot

The *residual demand* for firm *i* is the demand faces by firm *i*, taking the quantity output of all other firms as given. The idea is the follow: Since each firm takes the output of all other firm as given, it sees if it sell q_i units price would be

$$\begin{aligned} p &= P_D(q_1 + ... + q_{i-1} + q_i + q_{i+1} + ... + q_n) \\ &= a - bq_i - b(q_2 + ... + q_{i-1} + q_{i+1} + ... + q_n) \\ &= P_D(q_1) - b(q_2 + ... + q_{i-1} + q_{i+1} + ... + q_n) \end{aligned}$$

So effectively each firm is maximizing profit over a smaller left-over—*residual*— demand; the firms' behavior is identical to monopoly within the residual demand. This means the monopoly pricing rule applies,

III. Bertrand

Each seller chooses its price taking opponent's price as giving. Quantity determined by demand. Here we have a price war, resulting in perfectly competitive price.

Quantity:	Q^* given by $P_D(Q^*) = c$ Ant combination of $q_1 + q_2 = Q^*$ is solution
Price:	$p^* = c$

Output, **Price** and **Consumer Surplus** are the same as competitive market.
Profit is zero as in competitive market.

IV. Stackelberg

One seller gets to picks its quantity before the other. The main idea is that the seller who moves first has an advantage—it can take into consideration the other seller's response into consideration. The seller who moves later acts identically to a Cournot competitor; it takes the output of the first player as given when it gets to choose its own output.

Quantity::	$q_1^* = \frac{a-c}{2b}$ $q_2^* = \frac{a-c}{4b}$
Price:	$p^* = P_D(q_1^* + q_2^*)$

Individual Output for Seller 1 is higher than Cournot and lower for Seller 2.
Total Output is higher than Cournot and monopoly but lower than competitive market.
Price is lower than Cournot and monopoly but higher than competitive market
Consumer Surplus, as a result of above, is higher than Cournot but lower than competitive market
Individual Profit is higher than Cournot for Seller 1 but lower for Seller 2.
Total Profit is lower than Cournot and monopoly but higher than competitive market.

V. Example

Market Setting

$$P_D(Q) = 30 - 2Q$$
$$c = 4$$

Cournot

Bertrand

$$P_D(Q) = c$$
$$30 - 2Q = 4$$
$$Q = 13$$

so $q_1 + q_2 = Q = 13$

Price equals marginal cost = 4

Stackelberg

Always solve *first* the seller that moves *last*—this method is called *backward induction*. Seller 2's choice is exactly the same as in the Cournot case since q_1 has already been decided by Seller 1 when Seller 2 gets to choose. So from the Cournot case we know

$$q_2 = \frac{30 - 2q_1 - 4}{4}$$

Taking this into account, Seller 1 solves

$$= \max_{q_1}\{\pi_1\}$$
$$= \max_{q_1}\{P_D(q_1 + q_2)\cdot q_1 - c\cdot q_1\}$$
$$= \max_{q_1}\left\{\left[30 - 2\left(q_1 + \frac{30 - 2q_1 - 4}{4}\right)\right]\cdot q_1 - 4\cdot q_1\right\}$$

First order condition:

$$\frac{d}{dq_1}\left\{\left[30 - 2\left(q_1 + \frac{30 - 2q_1 - 4}{4}\right)\right]\cdot q_1 - 4\cdot q_1\right\} = 0$$

$$30 - 4q_1 - \frac{30}{2} + \frac{4}{2}q_1 + \frac{4}{2} - 4 = 0$$

$$q_1 = \frac{30 - 30/2 - 2}{2}$$
$$= \frac{13}{2}$$

Substitute this into the formula for q_2 gives $q_2 = \dfrac{30 - 2(13/2) - 4}{4} = \dfrac{13}{4}$

Which are exactly what the formulas give us.

Price is $P_D(q_1 + q_2) = 30 - 2(q_1 + q_2) = 30 - \left(\frac{13}{2} + \frac{13}{4}\right) = \frac{81}{4}$