University of California, Berkeley Spring 2007
ECON 100A Section 115, 116

# Functions, Marginal Analysis and Elasticities

## I. Office Hours

My office hours are Tuesday and Wednesday 12:30-1:30in 608-5 Evans. Whenever there is a problem set due on Tuesday I will have office hour same time, same place on Monday.

## II. Functions and a Simplified World View

In this class and economics as a whole we like to describe how things—people, factories or economy as a whole—react as functions. A function works like a factory—you give it inputs, it pumps out outputs. A typical example would be a *demand function*:

$$Q_D = D(P)$$

output → $Q_D$ ; input → $P$

where $P$ is price. Let us suppose this is the demand of a certain Joe for Golden Bears' hamburgers in a week. Given the price of GB's hamburgers, the demand function would be able to tell us how many GB's hamburgers Joe wants to buy.

In this class we will mainly deal with *linear* functions. I.e. functions that looks like

$$Q_D = 20 - 3P$$

Now you might say a person's demand for hamburgers of a certain restaurant should depend on her income, the price of other restaurants or a zillion other stuffs. That is true, but exactly mimicking the real world is not the point of economics. In economics we simplify the world into functions so to emphasis on key relationships. In the demand function's case, we want to emphasis on *the law of demand*—that quantity demanded decreases with price.

## III. THE RULE: Marginal Benefit = Marginal Cost

If there is one thing you should get out of this class besides the law of demand, it would be marginal benefit (MB) equals Marginal Cost (MC). This is the universal rule in economics for decision making, be it the purchase of hamburgers or stock market investment. Let us get started with two exercises on this rule. Marginal Benefit is the extra benefit one gets from doing the last unit of activity (e.g. the last hamburger Joe purchased); similarly, marginal cost is the extra cost one needs to pay from doing the last unit of activity.

Example 1

Orange, Inc. produces oPhone, a multimedia product.

| # of Units | Total Revenue (Benefit) | Marginal Benefit | Total Cost | Marginal Cost |
|---|---|---|---|---|
| 1 | 11 | 10 | 5 | 5 |
| 2 | 20 | 9 | 10 | 5 |
| 3 | 28 | 8 | 15 | 5 |
| 4 | 35 | 7 | 20 | 5 |
| 5 | 41 | 6 | 25 | 5 |
| 6 | 46 | 5 | 30 | 5 |
| 7 | 50 | 4 | 35 | 5 |

Mathematically, marginal benefit is the slope of total benefit, while marginal cost is the slope of total cost.

Example 2

Total revenue from selling x units of oPhone = TR(x) $= (11 - \frac{x}{2})x$

Total cost from selling x units of oPhone = TC(x) $= 5x$

Marginal Benefit = $\frac{dTR(x)}{dx} = \frac{d}{dx}\left[(11 - \frac{x}{2})x\right] = \frac{d}{dx}\left[11x - x^2/2\right] = 11 - x$

Marginal Cost = $\frac{dTC(x)}{dx} = \frac{d}{dx}\left[5x\right] = 5$

Marginal Benefit = Marginal Cost

11 – x = 5

x = 6

There is also another condition to check—that total benefit is bigger than total cost at MB = MC. It is very easy to verify in Example 1; you can also verify that this is true in example 2:

$$TR(6) = (11 - 6/2) * 6 = 48 > 30 = TC(6)$$

## IV. Elasticities

Elasticity describes the relationship between two variables. The definition of *the elasticity of B with respect to A* is the *percentage change in B with 1% change in A*. Mathematically,

$$\varepsilon_{B,A} = \frac{\text{\% change in B}}{\text{\% change in A}} = \frac{\%\Delta B}{\%\Delta A}$$

To be precise this is just an approximation—after all, it does not tell whether we should use the change from 1 unit of A to 2 units of A or 1 unit of A to 3 units of A; the results could be very different. To be precise we need to use calculus. Nevertheless when you are asked to use the above formula, always use the smallest change in unit possible (i.e. 1 to 2 units instead of 1 to 3 units).

Calculus version:

$$\varepsilon_{B,A} = \frac{\frac{dB}{B}}{\frac{dA}{A}} = \frac{A}{B}\frac{dB}{dA}$$

You have the formulas of common elasticities in lecture 1 slides. Here we will work on an example. Suppose Joe's demand of oPods is follows:

$$Q_{oPods} = Y - 3P_{oPods} + P_{Zane}$$

Where $Y$ is Joe's income, $P_{oPods}$ is the price of oPods and $P_{Zane}$ is the price of a competing product Zane.

Consider $Y = 20$, $P_{oPods} = 2$ and $P_{Zane} = 1$,
First find $Q_{oPods} = 20 - 3 \cdot 2 + 1 = 15$

1. (Own) price elasticity $= \frac{P_{oPods}}{Q_{oPods}}\frac{dQ_{oPods}}{dP_{oPods}} = \frac{2}{15}\cdot(-3) = -0.4$
2. Cross price elasticity $= \frac{P_{Zane}}{Q_{oPods}}\frac{dQ_{oPods}}{dP_{Zane}} = \frac{1}{15}\cdot 1 = 0.067$
3. Income elasticity $= \frac{Y}{Q_{oPods}}\frac{dQ_{oPods}}{dY} = \frac{20}{15}\cdot 1 = 1.33$